Modeling U.S. Historical Time-Series Prices and Inflation Using Alternative Long-Memory Approaches[ǂ]

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Abstract. We consider two important features of the historical U.S. price data (1774-2015), namely the data's persistence and cyclical structure. We first consider the persistence of the series, and focus on standard long-memory models that incorporate a peak at the zero frequency. We examine different models with respect to the deterministic terms, including non-linear deterministic trends of the Chebyshev form. Then, we investigate a more general model that includes both persistence and cyclicality of the series and, thus, includes two fractional integration parameters, one at the zero (long-run) frequency and the other at the non-zero (cyclical) frequency. We model the cyclical structure as a Gegenbauer process. This specification outperforms the standard long-memory specifications. We find that the order of integration at the zero frequency is about 0.5, and the one at the cyclical frequency is about 0.2 with cycles repeating approximately every 6 years, producing mean-reverting long-memory effects at both the zero and cyclical frequencies. Fitting the values to this model, however, we discover the presence of a break that, according to the methods employed, takes place at around 1940-41. The results indicate the prevalence of the long run or zero component with a much higher degree of persistence during the second post 1940-41 subsample, suggesting important implications for monetary policy.

Keywords: Persistence, Cyclicality, Chebyshev polynomials, Gegenbauer processes

JEL Classification: C22, E3

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1. Introduction

Most of the empirical literature on long-memory models of prices and inflation has focused on persistence, the case where the singularity or pole in the spectrum occurs at the zero frequency. A well-known measure of persistence is the fractional integration parameter at frequency zero, and different degrees of persistence, stationarity, and mean-reversion occur depending on the value of the fractional integration parameter (see, e.g., Gil-Alana, 2005a; Gadea and Mayoral, 2006; Kumar and Okimoto, 2007; Boubaker et al., 2017; Canarella and Miller, 2016, 2017a, 2017b).[1] In policy terms, the importance of persistence in prices and inflation stems from the economy's susceptibility to crisis and contagions as well as the possibility that exogenous shocks can produce permanent effects.

Persistence of prices and inflation at frequency zero, although a dominant characteristic of these time series, however, is not their only stochastic feature. Another feature present in many time series is stochastic cyclicality, that is, persistence at frequency away from zero. For instance, many macroeconomic time series, such as stock market prices, oil prices, and unemployment, exhibit dynamic characteristics where persistence at cyclical frequencies accompanies persistence at frequency zero (see, e.g., Gil-Alana, 2001; Gil-Alana and Gupta, 2014; Caporale and Gil-Alana, 2014). That is, the process exhibits both fractional integration at zero frequency and at a frequency away from zero (Gray et al., 1989, 1994). In this context,

[1] As the existing literature frequently notes, inflation persistence plays an important role in the conduct of monetary policy as well as the development of the underlying macroeconomic theories. Inflation persistence measures the speed with which the inflation rate returns to its equilibrium level after an inflationary shock. If the inflation rate returns to its equilibrium level quickly (i.e., the inflation rate exhibits less persistence) after a shock, then the monetary authorities can more effectively reduce inflation fluctuations, all else equal (Fuhrer, 1995). High inflation persistence, on the other hand, causes shocks to exert long-lasting effects and may require a strong policy response to affect the dynamics of inflation and bring it under control. In the worst case, inflation may follow a random-walk (i.e., I(1) process), making it impossible for central banks to control inflation. In the best case, inflation may follow a stationary (i.e., I(0) process), implying that it reverts to its equilibrium level rapidly after a random shock. In this latter case, the response to the inflationary shock may not require an active monetary policy. Thus, the optimal timing and size of monetary policy crucially depend on not only knowledge of how shocks affect the dynamics of inflation but also on the degree of persistence that identifies the inflation process. In this regard, we note that inflation persistence plays an important role in the current debate on inflation targeting. When a central bank successfully anchors inflationary expectations by its inflation targeting policy, it reduces or eliminates inflation persistence, since well-anchored inflationary expectations depend less on past inflation.

while the degree of fractional integration at frequency zero measures persistence and long-range dependence, the degree of fractional integration at a frequency away from zero indicates the degree of cyclical dependence.

One stylized fact that characterizes the economy over the business cycle is the co-movement of prices and output. If output movements result from demand shocks, prices are pro-cyclical; by contrast, if shocks originate from the supply side, prices are counter-cyclical. The new classical macroeconomics (Lucas, 1972, 1976) as well as Keynesian economics (Mankiw, 1989) provide evidence in support of a positive correlation between U.S. prices and output. The real business cycle theory, on the other hand, (Kydland and Prescott, 1982; Long and Plosser, 1983) supports the presence of an inverse relationship between prices and output. Whether prices exhibit pro-cyclical or countercyclical movement, the need to model adequately the cyclical component of prices is well documented in the literature.

This paper focuses on the estimation of the dual features of persistence and cyclicality in the historical series of the U.S. Consumer Price Index (CPI), spanning the period 1774 to 2015. The data cover the various components of the modern history of the international monetary systems, including the bimetallic standard era (1787-1873), the classical gold standard era (1873-1914), the interwar period (1915-1944), the Bretton Woods system (1945-1971), and the post-Bretton Woods system (1971-present) and, thus, provide a unique opportunity to consider how the time-series properties of U.S. prices vary across different monetary regimes and institutions. Clearly, over such a long time period, structural breaks probably have occurred between different regimes in price determination, and the empirical analysis should reflect such breaks. Consequently, in addition to persistence and cyclicality, this paper considers the possibility that nonlinearities may characterize the behavior of U.S. prices.

We estimate the U.S. data using a fractional integration approach, but employ a generalized definition of long-memory, which allows the inclusion of one or more singularities or poles in the spectrum at various frequencies. Specifically, we estimate U.S. prices with three classes of fractional integration I(d) models using the Whittle parametric function in the frequency domain (Dahlhaus, 1989) along with a Lagrange Multiplier (LM) testing procedure developed by Robinson (1994). The LM testing procedure proves the most efficient in the context of fractional integration against local alternatives, and remains valid even in nonstationary contexts.

The first class of models considers the standard case of fractional integration at the long-run or zero frequency, and captures the persistence of U.S. prices and inflation (i.e., the long-run movement at zero frequency). The most common approach uses the log-periodogram (e.g., Geweke and Porter-Hudak, 1983). This method was later extended and improved by many authors, including Phillips (2007) and Robinson (1995), among others. In this paper, we employ instead another semiparametric method, essentially a local 'Whittle estimator' defined in the frequency domain using a band of high frequencies that degenerates to zero (e.g., Caporale and Gil-Alana, 2002, 2007, 2010, 2013).

The second class adopts a fractional integration model that incorporates nonlinear deterministic terms in the form of Chebyshev time polynomials, as nonlinearities may exist in the historical data series as a result of different monetary regimes (Caporale and Gil-Alana, 2007). Several authors, such as Lee and Strazicich (2003), among others, have proposed unit-root tests incorporating structural breaks to improve the efficiency of the tests. Structural breaks, however, may still not adequately specify the deterministic component, as changes can occur smoothly rather than suddenly. Ouliaris et al., (1989) proposed regular polynomials to approximate the deterministic component of the data generating process. Bierens (1997) noted that Chebyshev polynomials, which are cosine functions of time, provide a better approximation of the deterministic component because of their orthogonality and boundedness.

Finally, the third class of long-memory models considers the possibility that the data may simultaneously display two poles or singularities in the spectrum, one at the zero frequency, corresponding to the long-run behavior of prices, and another at a frequency away from zero, affecting the cyclical structure of prices (Caporale and Gil-Alana, 2005; Gil-Alana, 2005; Caporale and Gil-Alana, 2014; Gil-Alana and Gupta, 2014). In this latter case, the data may still display the property of long-memory, but the autocorrelations exhibit a cyclical structure that decays slowly. Following the procedure due to Robinson (1994), we model the cyclical structure of the series as a Gegenbauer process, which produces persistent stochastic cycles. To the best of our knowledge, this is the first paper to combine the analysis of persistence and cyclicality of the historical series of the U.S. CPI in a single fractional integration framework.

We find that, though the root at zero frequency plays a much more important role (in terms of persistence) than the cyclical root, both the secular (long-run) and the cyclical components matter. The two orders of integration differ statistically from zero and one, implying that the cyclical frequency also displays long-memory behavior. Shocks affecting the two components persist and revert to their means (i.e., they disappear in the long run). Nevertheless, unlike the first two classes of long-term models, the analysis in the third class of models refers only to prices, and not inflation, since in first differences, the interaction with the cyclical component is not meaningful.

The paper’s outline includes the following sections. Section 2 briefly describes the various econometric methods. Section 3 reports the results of our econometric analysis applied to the full sample. Section 4 considers the problem of structural breaks. Since a break occurs around 1940, we repeat the analysis over the two subsamples. Section 5 briefly concludes.

2. Methods

All models examined rely on the concept of long-memory or long-range dependence as opposed to the concept of short-memory (i.e., I(0)) behavior. Since we can describe time-series analysis in the time or frequency domains, we can define both concepts (long memory and short memory) in the time and frequency domains. For short-memory processes, the time domain definition states that the infinite sum of the autocovariances γ_j is finite. That is,

$$\lim_{T \to \infty} \sum_{j=-T}^{T} \left|\gamma_j\right| < \infty.$$

In the frequency domain, short memory states that the spectral density function is defined as the Fourier transform of the autocovariances as follows:

$$f(\lambda) = \frac{1}{2\pi} \sum_{j=-\infty}^{\infty} \gamma_j \cos \lambda j, \qquad -\pi < \lambda \leq \pi,$$

which is positive and finite at all frequencies in the spectrum. That is,

$$0 < f(\lambda) < \infty \quad \text{for all} \quad \lambda \in [0, \pi).$$

Short-memory processes include the most common stationary process such as those based on (stationary) ARMA structures. In economics, however, it is common to find series that display a high degree of persistence, which we cannot capture using ARMA models. Thus, many economic series display long-memory behavior.

Hipel and McLeod (1978) define long-memory in the time domain as follows:

$$\lim_{T \to \infty} \sum_{j=-T}^{T} \left|\gamma_j\right| = \infty.$$

In the frequency domain, long memory implies that the spectral density function includes at least one pole or singularity at some frequency λ^* in the interval [0, π). That is,

$$f(\lambda) \to \infty, \quad as \ \lambda \to \lambda^*, \quad \lambda^* \in [0, \pi).$$

The empirical time-series literature usually focuses on the case where the singularity or spike in the spectrum takes place at the 0 frequency (i.e., $\lambda^* = 0$), which leads to the standard I(d) models of the form:

$$(1 - L)^{d_1} x_t = u_t, \quad t = 0, 1, ..., \tag{1}$$

where d_1 can equal any real value, L is the lag-operator (e.g., $Lx_t = x_{t-1}$), and u_t is I(0). If $d_1 = 0$, $x_t = u_t$ shows short-memory behavior. If $-0.5 < d_1 < 0.5$, the process is stationary. In particular, if $0 < d_1 < 0.5$, the process presents long memory. Instead, if $-0.5 < d_1 < 0$, the process is anti-persistent with short memory. If $0.5 \leq d_1 < 1$, the process is nonstationary, but still mean-reverting. The most notorious case corresponds to $d_1 = 1$, implying the existence of a unit root and non-stationarity. In this case, we need to transform by first differences to render the series I(0). This standard practice emerged after Nelson and Plosser (1982), who found evidence of unit roots in fourteen U.S. macro series. In general, however, the differencing of a series to achieve stationarity may, in fact, only require a fractional difference (Granger, 1980). As such, we identify the process as fractionally integrated. Then, we can expand the polynomial in the left-hand side of equation (1) in terms of its binomial expansion, such that, for all real d_1,

$$(1-L)^{d_1} = \sum_{j=0}^{\infty} \psi_j L^j = \sum_{j=0}^{\infty} \binom{d_1}{j} (-1)^j L^j = 1 - d_1 L + \frac{d_1(d_1-1)}{2} L^2 - ...,$$

or, equivalently,

$$(1-L)^{d_1} x_t = x_t - d_1 x_{t-1} + \frac{d_1(d_1-1)}{2} x_{t-2} - ...,$$

implying that we can express equation (1) as follows:

$$x_t = d_1 x_{t-1} - \frac{d_1(d_1-1)}{2} x_{t-2} + ... + u_t.$$

In this context, d_1 plays an essential role, since it defines the degree of dependence of the time series. The higher d_1 is, the higher is the level of association between the observations. Granger and Joyeaux (1980), Granger (1980, 1981), and Hosking (1981) introduced these models that Baillie (1996), Gil-Alana and Robinson (1997), and others later generalized them.

As previously noted, the existence of cycles in macroeconomic time series became a well-documented stylized fact after Burns and Mitchell (1946) first examined the U.S. economy. The appropriate way to model their cyclical behavior, however, remains controversial. Deterministic structures based on sine and cosine functions do not perform well in the majority of the cases. We can capture cyclical patterns through a simple AR(2) process with complex roots. In the case of high levels of persistence or even non-stationarity, however, a cyclical long-memory model can prove more appropriate. In such cases, we can extend the model in equation (1) by incorporating another pole or singularity in the spectrum at a non-zero frequency. Thus, we can represent x_t as follows:

$$(1-L)^{d_1}(1-2\cos w_r L + L^2)^{d_2} x_t = u_t, \quad t = 1, 2, \ldots, \tag{2}$$

where d_1 is the order of integration corresponding to the long-run or zero frequency, and d_2 is the order of integration with respect to the non-zero (cyclical) frequency, and u_t is an I(0) process.[2] The second polynomial in the left-hand side of equation (2) uses Gegenbauer processes, where $w_r = 2\pi r / T$ and $r = T/s$. Thus, s indicates the number of time periods per cycle, while r refers to the frequency with a pole or singularity in the spectrum of x_t. Note that if $r = 0$ (or $s = 1$), the fractional cyclical polynomial in equation (2) becomes $(1-L)^{2d_2}$, which is the polynomial associated with the long-run or zero frequency. Andel (1986) introduced this

[2]Hassler, et al. (2009) propose a similar procedure based on a LM test in the time domain to detect general forms of fractional integration at the long-run and/or the cyclical component of a time series.

process, which Gray et al., (1989, 1994), Giraitis and Leipus (1995), Chung (1996a, 1996b), Gil-Alana (2001) and Dalla and Hidalgo (2005) among others subsequently use.

Gray et al., (1989) show that, by denoting $\mu = \cos w_r$, one can express the second polynomial in equation (2) in terms of the orthogonal Gegenbauer polynomial $C_{j,d_2}(\mu)$ such that for all $d_2 \neq 0$,

$$(1 - 2\mu L + L^2)^{-d_2} = \sum_{j=0}^{\infty} C_{j,d_2}(\mu) L^j ,$$

where we can recursively define $C_{j,d_2}(\mu)$ as follows:

$$C_{0,d_2}(\mu) = 1, C_{1,d_2}(\mu) = 2\mu d_2, \text{and}$$

$$C_{j,d_2} = 2\mu\left(\frac{d_2 - 1}{j} + 1\right) C_{j-1,d_2}(\mu) - \left(2\frac{d_2 - 1}{j} + 1\right) C_{j-2d_2}(\mu), \ j = 2,3.., \ .$$

We estimate the fractional parameter *d*, which is a scalar in equation (1) but a (2x1) vector in equation (2), by different methods, including parametric and semiparametric ones. Moreover, we employ a Lagrange Multiplier (LM) test proposed by Robinson (1994) that allows x_t in equations (1) and (2) to equal the errors in a regression model of the form:

$$y_t = \beta^T z_t + x_t, \quad t = 1, 2, ..., \tag{3}$$

where y_t is the observed time series (e.g., log of US CPI), β is a (kx1) vector of unknown coefficients, and z_t is a set of strictly exogenous variables or deterministic terms that can include an intercept (i.e., $z_t = 1$), an intercept with a linear time trend ($z_t = (1, t)^T$), or any other type of deterministic process.

Thus, using equations (1) and (3), we can test the null hypothesis:

$$H_0 : d = d_0, \tag{4}$$

for any real value d_0, against the alternative $H_A: d \neq d_0$. Robinson (1994) shows that, under very general regularity conditions,[3] the limit distribution is χ^2 with one degree of freedom. Additionally, using Robinson's (1994) approach, we can test the null hypothesis:

$$H_0 : d \equiv (d_1, d_2)^T = (d_{10}, d_{20})^T \equiv d_0, \qquad (5)$$

in the model given by equations (2) and (3) for any given real values $d_0 = (d_{10}, d_{20})^T$, where T indicates the transposition operator. Robinson (1994) shows that the test statistic follows a χ^2 distribution with two degrees of freedom, and holds independently of the specification of the deterministic terms and strictly exogenous variables z_t and the modeling of the disturbances u_t. The specific forms of the test statistics appear in Robinson (1994) and also in Gil-Alana (2005). Unlike other procedures, this approach reduces to the classical large-sample testing methods. Several reasons exist for using this approach. Under Gaussianity, these tests prove the most efficient in the Pitman sense (i.e., requires fewer observations for inference at the same level of power) against local departures from the null. That is, if we implement them against local departures of the form $H_A : d = d_0 + \delta T^{-1/2}$, for $\delta \neq 0$, then the limit distribution is $\chi^2(\upsilon)$ with a non-centrality parameter υ that is optimal under Gaussianity of u_t. Moreover, we do not require Gaussianity for the implementation of this procedure, but only a moment condition of order 2.

In addition to this linear approach, we also employ an extension of this method to the nonlinear case, replacing the linear regression in equation (3) by a nonlinear model based on Chebyshev polynomials in time. Cuestas and Gil-Alana (2016) suggest this approach, which basically consists in replacing equation (3) by

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[3] These regularity conditions are rather mild, involving the behavior of u_t and specific technical assumptions on the two polynomials in equation (2).

$$y_t = \sum_{i=0}^{m} \theta_i P_{i,N}(t) + x_t \text{ , } t = 1, 2, \ldots \qquad (6)$$

where m gives the order of the Chebyshev polynomial $P_{i,N}(t)$, defined as,

$$P_{i,N}(t) = \sqrt{2}\cos[i\pi(t-0.5)/N]\,t = 1, 2, \ldots, N \text{ and } i = 1, 2, \ldots$$

with $P_{0,N}(t) = 1$. Bierens (1997) uses Chebyshev polynomials in the context of unit-root testing.

Chebyshev polynomials can approximate highly nonlinear trends with rather low degree polynomials (Bierens, 1997; Tomasevic and Stanivuk, 2009). From equation (16), if m = 0, the model contains only an intercept; if m = 1, it contains an intercept and a linear trend; and if m > 1, it becomes nonlinear, where higher values of m imply a more highly nonlinear structure.[4] The parameters $\theta_i\,(i = 1, \ldots, m)$ are the nonlinear parameters, where the significance of m > 1 parameters implies nonlinearity of the time series. An issue that immediately arises is the optimal value of m. Cuestas and Gil-Alana (2016) argue that if one combines equations (1) and (6) into a single equation, standard t-tests remain valid with an I(0) error term by definition. In other words, substituting equation (1) into equation (6), we obtain:

$$\tilde{y}_t = \sum_{i=0}^{m} \theta_i \tilde{P}_{i,N}(t) + u_t \text{ ,} \qquad (7)$$

where $\tilde{y}_t = (1-L)^{d_o} y_t$, $\tilde{P}_{i,N} = (1-L)^{d_o} P_{i,N}$, and d_0 is the value of d to be tested. Then, the choice of m will depend on the significance of the Chebyshev coefficients.[5] Note that the model obtained by combining equations (1) and (6) is linear, and we can estimate d parametrically and test this parameter as in Robinson (1994) and Demetrescu et al. (2008), among others (e.g., Cuestas and Gil-Alana, 2016).

3. Empirical results

[4] See Hamming (1973) and Smyth (1998) for a detailed description of these polynomials.

[5] See Cuestas and Gil-Alana (2016) for further details on the choice of *m*.

We gather the U.S. Consumer Price Index (CPI) data, covering the period 1774-2015, from the website of Professor Robert Sahr of Oregon State University,[6] and compute the inflation series as the first difference of the natural logarithm of the CPI, which implies that our effective sample starts from 1775.

Figure 1 shows the time-series plots of the log of CPI and the rate of inflation, along with their corresponding correlograms and periodograms. We observe that the CPI was relatively stable with some cyclical pattern until the Great Depression. After that, the CPI rose almost continuously until the present. We see the non-stationarity of the log CPI data through the correlogram, whose values decay slowly, and through the periodogram, whose highest value occurs at the smallest frequency. On the other hand, the correlogram of the inflation rate displays many significant values, while the periodogram also displays the highest frequency at the zero frequency. Nevertheless, this peak may hide others at a frequency away from zero.

[Insert Figure 1 about here]

We first examine the standard I(d) model. We estimate the parameters in equations (1) and (2) with $z_t = (1, t)^T$, and test the null $H_0 : d_1 = d_0$ for any real value d_0. That is,

$$y_t = \beta_0 + \beta_1 t + x_t ; \quad (1 - L)^{d_0} x_t = u_t . \tag{8}$$

We consider four different specifications of the error term u_t: white noise, AR(1), AR(2), and the nonparametric approach developed by Bloomfield (1973). The latter nonparametric method approximates ARMA structures with a few parameters and works well in fractional integration contexts (e.g., Gil-Alana, 2004). In particular, it is expressed exclusively in terms of its spectral density function defined as follows:

$$f(\lambda) = \frac{\sigma^2}{2\pi} \exp\left(\sum_{j=1}^{m} \tau_j \cos(\lambda j) \right), \tag{9}$$

[6] One can download the data from: http://liberalarts.oregonstate.edu/spp/polisci/research/inflation-conversion-factors.

where m is the order indicating the short-run dynamics.[7]

[Insert Table 1 about here]

Table 1 displays the estimates of d_1 along with the 95% confidence intervals of the non-rejection values of d_0 in equation (8) for both the log CPI and the inflation rate, and for the three standard cases examined in the literature of no regressors (i.e., $\beta_0 = \beta_1 = 0$ *a priori*): an intercept (i.e., β_0 unknown and $\beta_1 = 0$ *a priori*); and an intercept with a linear time trend (i.e., β_0 and β_1 unknown). We employ the Whittle function applied in the frequency domain as suggested by Dahlhaus (1989). The bolded entries in the table correspond to the most adequate specification for the deterministic terms, which according to the t-values of these coefficients, is the intercept-only case.[8] If u_t is white noise or follows the model of Bloomfield, then the estimated d_1 exceeds 1 and the unit-root null hypothesis (d_1 = 1) is, in fact, rejected in favor of the alternative of d_1 > 1. Using AR components, however, we cannot reject the unit-root hypothesis, even though the estimated d_1 still exceeds 1. Due to the disparity in these results, we also conducted various semi-parametric approaches (Robinson, 1995; Velasco, 1999; Abadir et al.,2007; Hou and Perron, 2014). In this context, we do not impose a functional form on the I(0) disturbance term.

[Insert Figure 2 and Table 2 about here]

Figure 2 displays the estimates of d_1 using the “local” Whittle method of Robinson (1995) and taking into account all the bandwidth values from m = 2, …, T/2. We observe that for small bandwidth values, the estimated values of d_1 lie within the I(1) interval, however, for large bandwidths, the values of d_1 are significantly above 1. Table 2 displays the specific values

[7]For the exponential spectral model of Bloomfield (1973), we tried different orders from 1 to 3. The results were similar in the three cases. Thus, we report the results only with m = 1.

[8]Similar to the nonlinear case above, expressing the two equations in (8) in a single equation produces I(0) errors, implying that t-values apply.

from m = 10 to 20 ($T^{0.5}$ = 15.55). We cannot reject the unit-root null hypothesis of d_1 = 1 in any single case. Performing alternative methods also based on the Whittle (Velasco, 1999; Abadir et al., 2007; Hou and Perron, 2014) produce essentially the same results.

The second model considers the possibility of nonlinear deterministic terms. For this purpose, we use the Chebyshev polynomials in time as presented in the previous section. Thus, the estimated model is specified as follows:

$$y_t = \sum_{i=0}^{m} \theta_i P_{iN}(t) + x_t; \quad (1-L)^{d_0} x_t = u_t \quad t = 1, 2, ..., \tag{10}$$

with m = 3. This choice was arbitrary; but, allowing higher values of m produced insignificant coefficients in all cases.

[Insert Table 3 about here]

We examine the cases of uncorrelated (white-noise) and autocorrelated (Bloomfield-type) errors. The results prove consistent in terms of the degree of integration. The estimated value of d_1 equals 1.27 for the log CPI data and 0.27 for the inflation rate with white-noise errors. These values are slightly smaller (1.12 and 0.11) for the Bloomfield-type disturbances and we cannot reject the unit-root null in these two cases. More importantly, we find evidence of nonlinearity in only a single case, corresponding to the log of CPI with autocorrelated errors.[9]

Finally, in the third model, we incorporate the possibility of cyclicality. Here, we consider a model of the following form:

$$y_t = \beta_0 + \beta_1 t + x_t; \quad (1-L)^{d_1}(1 - 2\cos w_r L + L^2)^{d_2} x_t = u_t, \quad t = 1, 2, ..., \tag{11}$$

and examine once more the three cases of no regressors, an intercept, and an intercept and a linear time trend, for the four cases of white noise, AR(1), AR(2), and Bloomfield-type

[9] Using other types of nonlinear deterministic terms such as Hermite polynomials does not produce any evidence of nonlinearities in the data.

disturbances. For each of these cases, we tried different values of $r = T/j$, with $j = 2, 3, \ldots, 19$ and 20, and choose the one that produces the lowest statistic using Robinson's (1994) tests. Table 4 displays the results.

[Insert Table 4 about here]

We observe that all the values of j (corresponding to the number of periods per cycle) fall between 5 and 13, which is consistent with the literature on business cycle. Moreover, except in the case of the AR(2) model, for the remaining models, the values of d_1 significantly exceed 1 with d_2 close to zero. Several tests based on the statistical significance of the deterministic terms and diagnostic tests carried out on the residuals, we conclude that the most appropriate model uses AR(2) disturbances with a linear time trend.[10]

Thus, the estimated model is as follows:

$$y_t \;=\; \underset{(14.294)}{1.95497} \;+\; \underset{(12.629)}{0.01182}\, t \;+\; x_t;$$

$$(1-L)^{0.54}(1-2\cos w_{T/6}\, L + L^2)^{0.21} x_t \;=\; u_t;$$

$$u_t \;=\; \underset{(6.631)}{0.542} u_{t-1} \;+\; \underset{(2.344)}{0.375} u_{t-2} \;+\; \varepsilon_t; \qquad R^2 \;=\; 0.761 \qquad (12)$$

with the t-values in parenthesis.

These findings clearly indicate that both the secular (i.e., the long-run) and the cyclical components matter. The two orders of integration differ statistically from zero and one, and the long-run order of integration appears more important (in terms of persistence). Shocks affecting the two components persist and revert to their means (i.e., they disappear in the long run).

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[10] In particular, we perform tests of no serial correlation, functional form, normality, and homoscedasticity using Microfit 5.0. For serial correlation, we use a Lagrange Multiplier test of residuals serial correlation (Godfrey,1978a,b): test statistic, 0.356; for the functional form, the Ramsey's (1969) RESET test using powers of the fitted values: test statistic, 1.145 and 1.177 with squared and cubic terms, respectively; for normality, a test based on skewness and kurtosis of residuals, (Bera and Jarque, 1981): test statistic, 3.490; and for homoscedasticity, we use Koenker (1981) modified LM test of Breusch and Pagan (1979): test statistic, 1.906.

We observe that in this case, the analysis can only refer to log CPI and not to inflation. That is, no direct way exists to derive the secular and cyclical persistence of inflation from the corresponding values of the persistence of log CPI. For inflation, we should conduct the analysis based on the first difference of log CPI. But if we take the first differences, the interaction with the cyclical component possesses no meaning, as the cyclical component disappears. Thus, the results imply that the two components matter only in the dynamic behavior of log CPI, and produce long-memory mean-reverting effects.

Despite these limitations, the results still have important implications for monetary policy. We show that the preferred model of log US CPI historical series incorporates both persistence and cyclicality. The findings clearly suggest that ignoring cyclicality is a serious misspecification, which, in turn, leads to an overestimation of the degree of persistence of log CPI. The price level does not follow unit-root dynamics when we account for persistence away from frequency zero. Shocks affecting the long-run component of the price level remains moderately persistent, while shocks affecting its cyclical component will disappear quickly. It follows that the monetary authorities should mainly focus on persistence of the price level at frequency zero.

[Insert Figure 3 about here]

Finally, one could argue that the analysis is simplistic, since it does not take into account alternative features of the data. In particular, the analysis does not consider the possibility that the CPI historical series includes structural breaks. Admittedly, this is a relevant issue. The use of Chebyshev polynomials play an important role in modeling the stability of the deterministic component, as changes can occur smoothly rather than suddenly. Analysis of structural breaks is a complex issue, and the interplay between the secular and cyclical components in the third model complicates the analysis further, as structural breaks may occur in both d_1 and d_2 or only in one or the other. The issue is further complicated by the fractional integration approach, since we can easily confuse long-memory processes with regime switching processes. It is well-known that fractional integration may disguise structural breaks, and vice versa. A large literature is developing on long memory and structural breaks (e.g., Bos et al., 1999; Diebold and Inoue, 2001; Granger and Hyung, 2004; Gil-Alana, 2007; and Andre et al., 2014). Discriminating between the two processes may prove difficult, since fractional integration may result as an artificial artefact of structural breaks not accommodated in the models (e.g., Ben Nasr, et al. 2014). Thus, it seems sensible to check the robustness of the results to structural change. In Figure 3, we plot the original data and the estimated values according to the latest specification given by (12). We clearly observe that at least one structural break may be missing from the model. Because of that, in the following section, we consider the presence of structural breaks from an empirical viewpoint.

4. The possibility of breaks

In this section, we consider the possibility of structural breaks in the data. Note that fractional integration and structural breaks are intimately related (e.g., Diebold and Inoue ,2001, Granger and Hyung, 2004, among others). Based on the long span of data used in this application, this becomes a relevant issue.

First, we employ Bai and Perron's (2003) method on the log CPI data and the results suggest the existence of a single break near 1941. Next, we conduct Gil-Alana's (2008) approach, which is specifically designed for the case of fractional integration. The results, which prove consistent with Bai and Perron (2003) and Hassler and Meller (2004) indicate the existence of a break date around 1940-41. Thus, we separate the sample into two subsamples around such a break date and re-estimate each of the models discussed in Section 3 over the two subsamples.

We start with the model containing a single pole or singularity at the zero frequency. That is, we consider the model given by equation (8). Table 5 considers the parametric approach and reports the estimates of d_1 for each subsample under the four modeling assumptions for the error term (i.e., white noise, AR(1), AR(2) and Bloomfield). The findings indicate that the presence of a structural change affects the results obtained for the whole sample. First, we observe that the time trend is only required in the second subsample, the intercept being sufficient to describe the deterministic term in the first subsample. Second, we observe higher orders of integration in the second subsample. Thus, with the data ending at 1940, the unit-root hypothesis ($d_1 = 1$) is rejected in favor of $d_1 > 1$ for uncorrelated errors, but cannot be rejected in the three remaining cases. For the second subsample, however, the estimates are significantly above 1 under white noise, AR(1), and Bloomfield-type errors. For the AR(2), the estimated value of d_1 equals 1.55, but the confidence interval is so wide that we cannot reject the unit-root null.

[Insert Tables 5 and 6 about here]

Table 6 displays the estimates of d_1 based on the semiparametric Whittle approach. The results prove very conclusive in favor of mean reversion ($d_1 < 1$) during the first subsample, but even the unit-root hypothesis is rejected in favor of $d_1 > 1$ after the 1940 date break.

We finally re-estimate over the two subsamples the model with two potential poles in the spectrum, one at the zero frequency and the other one at a non-zero (cyclical) one. Table 7 displays the results. The most noticeable feature of the findings is that the fractional differencing parameter corresponding to the cyclical frequency become insignificant in all cases. This outcome probably is the consequence of the large confidence intervals due to the smaller sample sizes. Thus, though the values of j (number of cycles per cycle) are higher during the first subsample, they are not reliable based on the insignificance of the d_2-coefficients. Considering the long run parameter, however, the results are consistent with those in Tables 5 and 6, in particular the finding of higher degrees of persistence during the second subsample.

5. Concluding remarks

This paper analyzes the complete historical U.S. price data (1774-2015) using a variety of model specifications that incorporate the concept of long memory, persistence, nonlinearity, cyclicality, and structural change. We estimate U.S. CPI with three classes of fractional integration, I(*d*), models using the Whittle parametric function in the frequency domain (Dahlhaus, 1989) along with the testing procedure developed by Robinson (1994). We consider, in addition to the well-known linear specifications at zero frequency, the possibility of nonlinear deterministic trends as well as the possibility that persistence exists at both the zero frequency and a frequency away from zero. We model the fractional nonlinear case using Chebyshev polynomials and model the fractional cyclical structures as a Gegenbauer process. We find evidence of nonlinearity in only a single case, corresponding to the inflation rate with white-noise errors.

The first important contribution of the paper consists of the determination of persistence at frequencies away from zero. We find that the secular (i.e., long-run) persistence coexists with the cyclical persistence, and shocks cause long-memory effects that revert to the mean at both the long-run and cyclical frequencies**.** We find that the two orders of fractional integration differ statistically from zero and one, with the secular order of fractional integration higher and, consequently, more important in terms of persistence, than the cyclical order.

The second important contribution of the paper is that the presence of a structural break in the context of fractional integration methods is an extremely complex issue. We conduct both Bai and Perron (2003) and Gil-Alana (2008) tests, the latter specifically designed for the analysis of structural breaks with I(*d*) data. The results of both tests indicate the presence of a single break at around 1940. The findings of the separate estimation of each subsample show the prevalence of the integration order at the long run frequency, since the cyclical order of integration proves statistically insignificant in all cases. Moreover, higher degrees of persistence exist in all cases during the second subsample.

Finally, an issue not fully developed in this paper concerns the appropriate model specification across the different models presented. Some models are nested and likelihood ratio tests determine the appropriate ones. In other cases, however, the models are not nested and we cannot use LR tests. Nevertheless, our initial results support the specification based on both secular and cyclical persistent components. After considering the role of structural change, however, our findings reduce the role of the cyclical component. In sum, our overall findings support Federal Reserve policy that focuses on issues of the long-run trend in the data and not short-run cyclical effects. This proves most consistent with the traditional monetarist school of thought.

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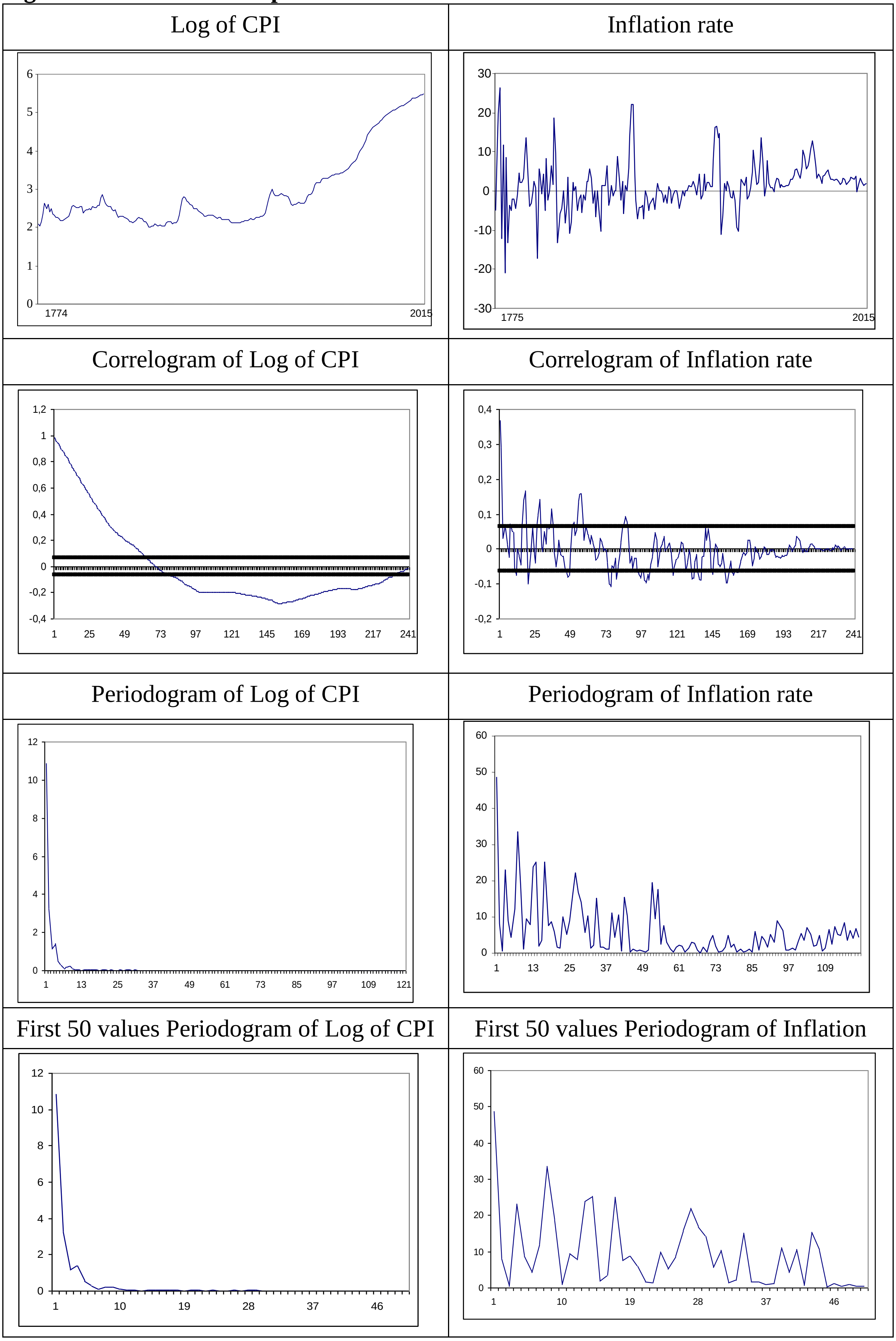
Figure 1: Time series plots



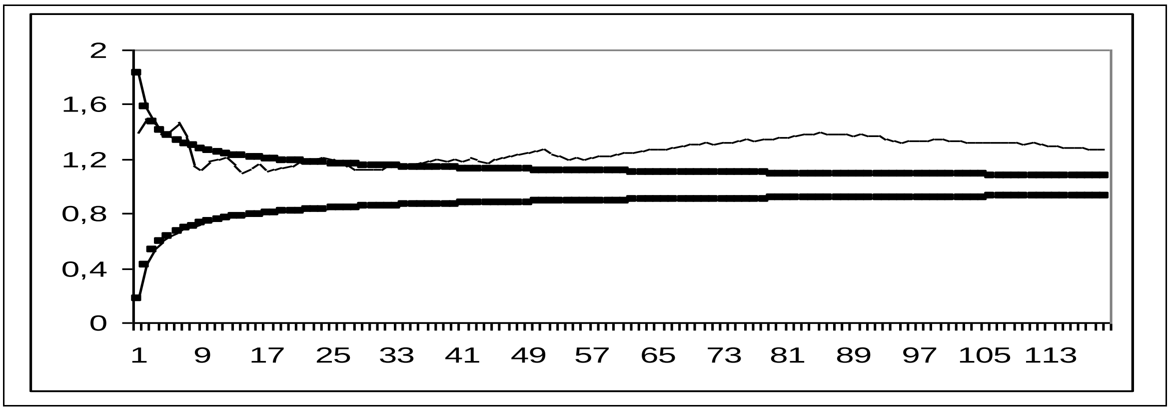
Notes: The thick lines in the correlogram indicate the bands for no autocorrelation at the 5% level.

Table 1: Estimates of d_1 and the 95 percent confidence interval using a parametric method

i) Log of CPI			
	No regressors	An intercept	A linear time trend
White noise	1.06 (0.99, 1.15)	**1.29 (1.20, 1.41)**	1.29 (1.20, 1.41)
AR (1)	1.41 (1.26, 1.59)	**1.13 (0.92, 1.47)**	1.15 (0.91, 1.48)
AR (2)	1.92 (1.71, 2.14)	**1.02 (0.85, 1.31)**	1.02 (0.82, 1.32)
Bloomfield type	1.13 (1.00, 1.33)	**1.21 (1.08, 1.41)**	1.22 (1.09, 1.42)
ii) Inflation			
	No regressors	An intercept	A linear time trend
White noise	0.29 (0.20, 0.41)	**0.29 (0.20, 0.41)**	0.28 (0.18, 0.41)
AR (1)	0.13 (-0.08, 0.49)	**0.15 (-0.01, 0.48)**	0.16 (-0.02, 0.48)
AR (2)	0.01 (-0.14, 0.31)	**0.01 (-0.15, 0.32)**	0.01 (-0.14, 0.33)
Bloomfield type	0.21 (0.08, 0.42)	**0.21 (0.09, 0.41)**	0.14 (-0.03, 0.40)

Notes: The 95% confidence intervals of non-rejection of the values of d_1 using Robinson's (1994) parametric approach appear in parentheses. Bolded numbers identify the selected specifications.

Figure 2: Estimates of d_1 based on a semiparametric method (Robinson, 1995)



Notes: In bold lines, the 95% confidence of the I(1) hypothesis (i.e., d_1 = 1).

Table 2: Robinson's (1995) estimates of d_1 for the log CPI

m	d_1	Lower 5%	Upper 5%
10	1.186*	0.739	1.260
11	1.194*	0.752	1.247
12	1.206*	0.762	1.237
13	1.143*	0.771	1.228
14	1.099*	0.780	1.219
15	1.133*	0.787	1.212
16	1.160*	0.794	1.205
17	1.115*	0.800	1.199
18	1.126*	0.806	1.193
19	1.133*	0.813	1.188
20	1.147*	0.816	1.184

Notes: The *m* values indicate the bandwidth number.
* indicates evidence of I(1) behavior at the 95% level.

Table 3: Estimates of the nonlinear coefficients and *d* using Cuestas and Gil-Alana (2016)

i) Log of CPI					
	d_1 (95% interval)	θ_1	θ_2	θ_3	θ_4
Wh. Noise	1.27 (1.17, 1.40)	2.3655 (1.42)	-0.5335 (-0.50)	0.5413 (1.37)	-0.2069 (-0.87)
Bloomfield	1.12 (0.95, 1.28)	2.6075 (3.11)	-0.6920 (-1.33)	0.5557 (2.46)	-0.2423 (-1.69)
ii) Inflation					
	d_1 (95% interval)	θ_1	θ_2	θ_3	θ_4
Wh. Noise	0.27 (0.15, 0.49)	1.5102 (1.01)	-1.1386 (-1.06)	0.7216 (0.75)	0.4944 (0.75)
Bloomfield	0.11 (-0.13, 0.37)	1.4252 (2.80)	-1.2558 (-2.22)	0.6940 (1.28)	0.4345 (0.83)

Notes: The values in parentheses in the second column refers to the 95% confidence intervals for the values of d_1. In the remaining columns, *t*-values appear in parentheses.

Figure 3: Original values and estimated values on the log CPI

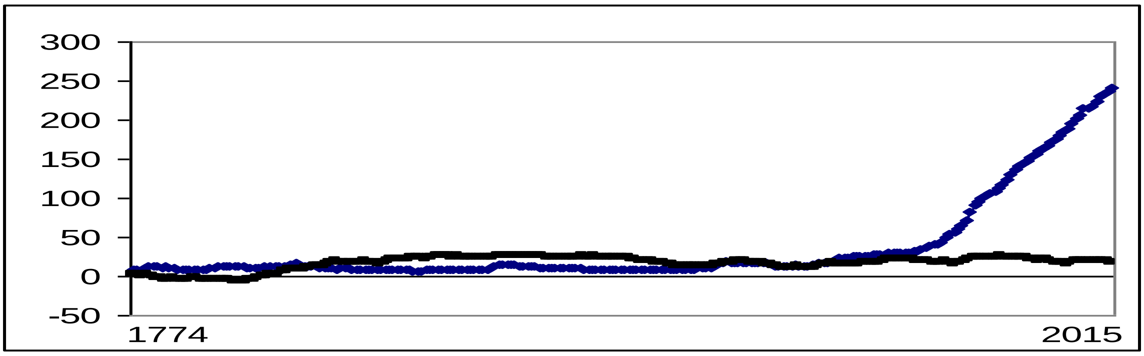


Table 4: Estimates of the long-run and cyclical persistence parameters in the model given by (11) for the log CPI

	Determ. terms	j	d_1	d_2
White noise	No terms	5	1.34 (1.23, 1.46)	-0.05 (-0.10, 0.09)
	An intercept	7	1.29 (1.21, 1.39)	0.00 (-0.07, 0.08)
	A linear trend	7	1.29 (1.21, 1.47)	0.01 (-0.07, 0.08)
AR(1)	No terms	8	1.33 (1.02, 1.60)	0.02 (-0.29, 0.26)
	An intercept	9	1.19 (1.12, 1.37)	0.07 (-0.04, 0.34)
	A linear trend	9	1.26 (1.14, 1.40)	0.21 (-0.01, 0.37)
AR(2)	No terms	6	0.78 (0.69, 0.93)	-0.35 (-0.41, -0.29)
	An intercept	6	0.92 (0.83, 1,05)	-0.44 (-0.58, -0.32)
	A linear trend	**6**	**0.54 (0.27, 0.83)**	**0.21 (0.04, 0.41)**
Bloomfield	No terms	13	1.48 (1.11, 1.54)	-0.06 (-0.38, 0.14)
	An intercept	9	1.19 (1.03, 1.40)	0.07 (-0.32, 0.16)
	A linear trend	8	1.24 (1.10, 1.43)	0.09 (-0.08, 0.25)

Notes: The 95% confidence intervals are given in parentheses next to the estimates of d_1 and d_2. Bolded numbers identify the selected specifications.

Table 5: Estimates of d_1 for each subsample in the log CPI using a parametric method

i) First subsample (1774 – 1940)			
	No regressors	An intercept	A linear time trend
White noise	1.13 (1.02, 1.27)	**1.21 (1.08, 1.38)**	1.21 (1.08, 1.38)
AR(1)	1.18 (1.01, 1.58)	**1.07 (0.92, 1.39)**	1.07 (0.91, 1.39)
AR(2)	1.22 (1.10, 1.63)	**0.97 (0.63, 1.27)**	0.94 (0.49, 1.28)
Bloomfield	1.06 (0.87, 1.37)	**1.00 (0.76, 1.34)**	1.00 (0.76, 1.34)
ii) Second subsample (1941 – 2015)			
	No regressors	An intercept	A linear time trend
White noise	1.32 (1.22, 1.50)	1.49 (1.36, 1.74)	**1.54 (1.42, 1.76)**
AR(1)	1.22 (1.04, 1.61)	1.30 (1.16, 1.49)	**1.38 (1.17, 1.56)**
AR(2)	1.38 (0.96, 1.97)	1.47 (0.79, 2.03)	**1.55 (0.84, 2.03)**
Bloomfield	1.24 (1.12, 1.44)	1.28 (1.16, 1.46)	**1.36 (1.22, 1.55)**

Note: The 95% confidence intervals are given in parentheses next to the estimate of d_1. Bolded numbers identify the selected specifications.

Table 6: Estimates of *d* for each subsample in the log CPI using a semiparametric method

m	First subsample	Second subsample	Lower 5%	Upper 5%
10	0.686*	1.500**	0.739	1.260
11	0.762	1.500**	0.752	1.247
12	0.736*	1.500**	0.762	1.237
13	0.779*	1.362**	0.771	1.228
14	0.829	1.304**	0.780	1.219
15	0.898	1.362**	0.787	1.212
16	0.936	1.346**	0.794	1.205
17	0.989	1.262**	0.800	1.199
18	0.957	1.266**	0.806	1.193
19	0.957	1.277**	0.813	1.188
20	0.813*	1.296**	0.816	1.184

Note: The first and second subsamples run from 1774–1940 and 1941–2015, respectively.
* significantly less than one at the 5% level.
** significantly greater than one at the 5% level.

Table 7: Estimates of the coefficients in equation (11) for each subsample

i) First subsample (1774 – 1940)				
	Determ. terms	j	d_1	d_2
White noise	No terms	10	0.43	0.10
	An intercept	8	0.98*	0.14
	A linear trend	8	0.97*	0.15
AR(1)	No terms	11	0.95*	-0.07
	An intercept	11	0.99*	0.05
	A linear trend	11	0.99*	0.10
AR(2)	No terms	10	0.67*	0.09
	An intercept	10	0.68*	0.05
	A linear trend	9	0.79*	0.07
Bloomf.	No terms	9	0.60*	0.01
	An intercept	9	0.58*	-0.03
	A linear trend	10	0.62*	0.04
i) Second subsample (1941 – 2015)				
	Determ. terms	J	d_1	d_2
White noise	No terms	7	1.17*	0.04
	An intercept	5	1.16*	0.30
	A linear trend	5	1.17*	0.26
AR(1)	No terms	5	1.33*	-0.05
	An intercept	6	1.36*	-0.06
	A linear trend	6	1.25*	-0.02
AR(2)	No terms	6	1.23*	0.02
	An intercept	6	1.23*	-0.02
	A linear trend	7	1.09*	0.04
Bloomf.	No terms	7	1.33*	-0.06
	An intercept	7	1.34*	-0.06
	A linear trend	6	1.35*	-0.04

* estimates of d_1 and d_2 significantly different from 0 at the 5% level.