The Great Moderation and Leptokurtosis after GARCH Adjustment

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Abstract

Recently, Fagiolo *et al.* (2008) find fat tails of economic growth rates after adjusting outliers, autocorrelation and heteroskedasticity. This paper employs US quarterly real output growth, showing that this finding of fat tails may reflect the Great Moderation. That is, leptokurtosis disappears after GARCH adjustment once we incorporate the break in the variance equation.

Keywords: real GDP growth, the Great Moderation, leptokurtosis, GARCH models

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1. Introduction

In a recent study, assuming time invariance of the underlying generating mechanism governing output growth dynamics, Fagiolo *et al.* (2008) find that fat tails characterize output growth rate distribution after adjusting outliers, autocorrelation and heteroskedasticity for the US and other OECD countries. Using US quarterly real GDP growth rate as an example, we show that once we take account of the Great Moderation and thus incorporate a structural break in the variance equation, fat tails of output growth disappear in a simple autoregressive generalized autoregressive conditional heteroskedasticity (AR-GARCH) model, either under a symmetric or an asymmetric specification.

The AR-GARCH(1,1) model

$$y_t = a_0 + \sum_{i=1}^{p} a_i y_{t-i} + \varepsilon_t , \tag{1}$$

$$\varepsilon_t = \sigma_t \eta_t , \quad \sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 , \tag{2}$$

where $\eta_t \sim$ iid(0,1), introduced by Bollerslev (1986) is frequently employed to model excess kurtosis and volatility clustering in the output growth rate y_t .

The model contained in equations (1) and (2) assumes that positive and negative shocks generate the same effect on volatility, implying a symmetric GARCH specification. The volatility may respond differently to shocks during periods of a rise or a fall in output growth. To provide a systematic analysis, we also examine the exponential GARCH (EGARCH) process introduced by Nelson (1991) as follows:

$$log\ \sigma_t^2 = \alpha_0 + \alpha_1 \frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} + \alpha_2 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_1\ log \sigma_{t-1}^2 , \tag{3}$$

where asymmetry exists if $\alpha_2 \neq 0$, and when $\alpha_2 < 0$, it implies that negative shocks generate higher volatility than positive shocks of the same magnitude, and vice versa. The log transformation guarantees a positive variance.

Real GDP growth involves long-run phenomena. For a longer sample period, structural changes in the mean and variance in addition to outliers will occur with a higher probability. Although no evidence of changes in the mean growth exists in studies, McConnell and Perez-Quiros (2000) among others, document a structural change and find a rather dramatic reduction in the volatility of US GDP growth that some have labeled the Great Moderation, which violates time invariance assumed by Fagiolo *et al.* (2008).

In this study, following Franses and Ghijsels (1999), we first apply the method of Chen and Liu (1993) to detect and correct for additive outliers in the GARCH models. We then use Bai and Perron's (1998, 2003) multiple structural change test to identify breaks in the mean and variance of the growth rate, if any. Finally, we incorporate the changes in the GARCH models to observe the behavior of leptokurtosis. In the application, we provide new evidence that leptokurtosis disappears when the break enters into the variance equation. That is, the finding of fat tails of the growth rate is not robust to the structural change in the variance, or the Great Moderation, which, however, is commonly recognized by researchers.

2. Data and Empirical Results

Output growth rates (y_t) equal the percentage change in the logarithm of seasonally adjusted quarterly real GDP, that equals nominal GDP deflated by the GDP deflator with base year 2000. All data come from the IMF *International Financial Statistics* (*IFS*) over the period 1957:1 to 2008:1.

We find an additive outlier at the date 1978:2, which is the maximum observation in the original growth series. Table 1 reports summary statistics for the outlier-corrected growth rate. In Panel A, the skewness statistic displays an asymmetric distribution. The negative skewness means that in the sample period, a greater probability exists of large decreases in real GDP growth than larger increases. The kurtosis statistic exhibits leptokurticity with fat tails. A higher kurtosis means that extreme changes occur more frequently. The Jarque-Bera test rejects normality. The Ljung-Box statistics for the growth rates ($LB\ Q$) and its squared rates ($LB\ Q^2$) indicate autocorrelation and heteroskedasticity up to six lags, suggesting ARMA processes for the mean and the variance equations to capture the dynamic structure and to generate white-noise residuals.

We construct an AR model for the mean growth rate. The SBC selects AR(2) and Panel B shows that this process proves adequate to produce uncorrelated residuals. Skewness, kurtosis, and heteroskedasticity, however, remain in residuals. GARCH-type process is applied to capture the time-varying variance. We estimate the GARCH(1,1) models employing Bollerslev and Wooldridge's (1992) quasi-maximum likelihood estimation technique, assuming normally distributed errors and using the BHHH algorithm.

In Panels C and D, the fitted GARCH and EGARCH models adequately capture the time-series properties of the growth rate in that the Ljung-Box Q-statistics for standardized residuals and standardized squared residuals, up to 6 lags, do not detect autocorrelation and heteroskedasticity. The residuals, however, exhibit significant leptokurtosis for the two models at least at 10-percent significance level. Another cautionary note is that the likelihood ratio (LR) tests for $\alpha_1 + \beta_1 = 1$ in the GARCH and $\beta_1 = 1$ in the EGARCH process do not reject the null hypothesis of an integrated GARCH (IGARCH) effect. The high persistence measures may reflect structural changes in the mean or variance of the growth rate, which the GARCH estimations ignore (Diebold, 1986; Hillebrand, 2005; and Krämer and Azamo, 2007).

In sum, the fat tails of the growth rate exists after we adjust an outlier, autocorrelation, and heteroskedasticity in a symmetric or an asymmetric GARCH model, ignoring the Great Moderation.

The Bai and Perron's (1998, 2003) test finds no change in the mean. Table 2 displays the results of testing for breaks in the variance, their critical values at the 5-percent significance level (in parentheses), and structural stability test. In Panel A, the $\sup F(5|0)$ test proves significant for $m=5$, suggesting the existence of at least one break in the variance. The two double maximum statistics, UD_{max} and WD_{max}, agree with this result. The test, $\sup F(2|1) = 1.6396$ falls below the critical value, suggesting that a single structural break exists at 1984:1 with 95% confidence interval [1982:4-1989:3], the same break date found by McConnell and Perez-Quiros (2000). In Panel B, we further conduct a variance-ratio statistic test for the equality of the unconditional variances by splitting the sample into sub-periods according to the break date. A clear decline in the standard deviation of the growth rate occurs from 1.1040 in the pre-1984 sample to 0.4934 in the post-1984 sample. The p-values for the variance-ratio F-test significantly reject the null of variance equality between the samples. The decline equals 55-percent. As the introduction notes, economists call the substantial drop in the variance of output growth in the period after the break the Great Moderation.

To examine the effect of the Great Moderation on leptokurtosis, we include a dummy variable in the conditional variance equation, which equals unity from the break date forward, zero otherwise, in the GARCH and EGARCH processes, respectively, as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma D, \qquad (4)$$

$$log\ \sigma_t^2 = \alpha_0 + \alpha_1 \frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} + \alpha_2 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_1\ log \sigma_{t-1}^2 + \gamma\ D, \quad (5)$$

where D = 1 for $t > 1984:1$; and 0, otherwise. The dummy variable accommodates the extraordinary change. Since the volatility declines, we expect a significant negative estimate of γ to capture the break in the variance process.

Table 3 reports the estimates with the variance dummy variable. The coefficient of the structural dummy (i.e., γ) proves significantly negative in the variance equation at the 5-percent level. The Ljung-Box Q-statistics show no evidence of autocorrelation and heteroskedasticity. The coefficients of skewness and kurtosis prove insignificant. Thus, the residuals conform to a normal distribution. For both the symmetric and the asymmetric GARCH models, these results suggest that the statistical evidence for leptokurtosis in the growth rate may reflect structural change in the variance caused by the Great Moderation. Additionally, the significant LR statistics indicate no IGARCH effect. High volatility persistence also reflects the Great Moderation.

3. Conclusion

Using GARCH modeling approach, we show that fat tails of US quarterly real GDP growth rates exists before and after the adjustments of outliers, autocorrelation, and heteroskedasticity under time invariant assumption of the variance. Instability or the Great Moderation, however, governs the variance process. Once we incorporate the break into the variance equation, fat tails in the GARCH residuals disappear. This completes the unfinished tale of leptokurtosis of the output growth rate as told by Fagiolo *et al.* (2008).

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Table 1: Summary Statistics for Quarterly Real GDP Growth, 1957-2008

Panel A. Descriptive Statistics

Mean	**Standard deviation**	**Maximum**	**Minimum**	**Skewness**	**Kurtosis**	**Normality test**
0.7831	0.8703	2.7320	-2.7528	-0.5644*	1.4517*	28.7443*
				[0.0010]	[0.0000]	[0.0000]
***LB Q* (1)**	***LB Q* (2)**	***LB Q* (3)**	***LB Q* (4)**	***LB Q* (5)**	***LB Q* (6)**	
19.8267*	27.3615*	27.7672*	27.7695*	32.4020*	32.9619*	
[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	
***LB* Q^2 (1)**	***LB* Q^2 (2)**	***LB* Q^2 (3)**	***LB* Q^2 (4)**	***LB* Q^2 (5)**	***LB* Q^2 (6)**	
8.8586*	16.9030*	20.6155*	21.9712*	22.3580*	22.3802*	
[0.0029]	[0.0002]	[0.0001]	[0.0002]	[0.0004]	[0.0010]	

Panel B. AR(2) Estimates

a_0	a_1	a_2		
0.4836*	0.2799*	0.1042		
(0.0875)	(0.0704)	(0.0703)		
***LB Q* (6)**	***LB* Q^2 (6)**	**Skewness**	**Kurtosis**	**Normality**
5.8688	67.2174*	-0.4270*	1.2088*	18.4379*
[0.4380]	[0.0000]	[0.0139]	[0.0005]	[0.0000]

Panel C. GARCH(1,1) Estimates

a_0	a_1	a_2			
0.4511*	0.2581*	0.1930*			
(0.0846)	(0.0786)	(0.0761)			
α_0	α_1	β_1			
0.0301*	0.1728*	0.7796*			
(0.0148)	(0.0820)	(0.0922)			
***LB Q* (6)**	***LB* Q^2 (6)**	**LR**	**Skewness**	**Kurtosis**	**Normality**
5.9911	4.8385	1.8438	-0.4285*	1.2492*	19.3176*
[0.4241]	[0.5646]	[0.1761]	[0.0135]	[0.0003]	[0.0000]

Panel D. EGARCH(1,1) Estimates

a_0	a_1	a_2			
0.4684*	0.1946*	0.2202*			
(0.0955)	(0.0792)	(0.0740)			
α_0	α_1	α_2	β_1		
-0.2908*	0.2934*	-0.0938	0.9305*		
(0.0966)	(0.0589)	(0.0589)	(0.0494)		
***LB Q* (6)**	***LB* Q^2 (6)**	**LR**	**Skewness**	**Kurtosis**	**Normality**
4.8058	7.8057	1.9743	-0.2125	0.6329**	4.7967**
[0.5689]	[0.2526]	[0.1616]	[0.2255]	[0.0740]	[0.0908]

Note: p-values appear in brackets; 0.0000 indicates less than 0.00005. The measures of skewness and kurtosis are normally distributed as $N(0,6/T)$ and $N(0,24/T)$, respectively, where T equals the number of observations. In Panel A, $LB\ Q(k)$ and $LB\ Q^2(k)$ equal Ljung-Box Q-statistics, testing for level and squared terms for autocorrelations up to k lags. In Panel B, C and D, standard errors appear in parentheses; $LB\ Q(k)$ and $LB\ Q^2(k)$ test for (standardized) residuals and squared (standardized) residuals for autocorrelations up to k lags, where the degrees of freedom are reduced by the number of estimated coefficients in the mean equation. LR equals the likelihood ratio statistic, following a χ^2 distribution with one degree of freedom that tests for $\alpha_1 + \beta_1 = 1$ in GARCH and $\beta_1 = 1$ in EGARCH, respectively.

* denotes 5-percent significance level.

** denotes 10-percent significance level.

Table 2: Break Date and Structural Stability Test

Panel A. Structural Break Test in Variance

$Sup\ F(1\|0)$	$Sup\ F(2\|0)$	$Sup\ F(3\|0)$	$Sup\ F(4\|0)$	$Sup\ F(5\|0)$	UD_{max}	WD_{max}
44.8692*	23.0062*	15.6404*	12.3835*	11.1895*	44.8692*	44.8692*
(8.5800)	(7.2200)	(5.9600)	(4.9900)	(3.9100)	(8.8800)	(9.9100)
$Sup\ F(2\|1)$	$Sup\ F(3\|2)$	$Sup\ F(4\|3)$	$Sup\ F(5\|4)$	Break date	95% Confidence Interval	
1.6396	1.0154	0.7364	0.7119	1984:1	[1982:4-1989:3]	
(10.1300)	(11.1400)	(11.8300)	(12.2500)			

Panel B. Structural Stability Test

Break date	**Period**	**Mean**	**Standard Deviation**	**Sub-sample 1 v.s. Sub-sample 2**
1984:1	1957:1-1984:1	0.8130	1.1040	5.0059*
	1984:2-2008:1	0.7494	0.4934	[0.0000]

Note: Critical values for the 5-percent significance level appear in parentheses. P-values appear in brackets; 0.0000 indicates less than 0.00005. *F* test equals the unconditional variance ratio test between the samples *i* and *j*, and is asymptotically distributed as $F(df_i, df_j)$, where *df* denotes the degrees of freedom.

* denotes 5-percent significance level.

Table 3: GARCH(1,1) Estimates with Structural Break in Variance

Panel A. GARCH(1,1) Estimates

a_0	a_1	a_2			
0.4638*	0.2139*	0.1770*			
(0.0793)	(0.0714)	(0.0715)			
α_0	α_1	β_1	γ		
0.9306*	0.0981	0.1004	-0.7754*		
(0.3287)	(0.0899)	(0.2656)	(0.2801)		
LB Q **(6)**	***LB*** Q^2 **(6)**	**LR**	**Skewness**	**Kurtosis**	**Normality**
2.5908	5.1249	9.2517*	-0.1629	-0.1508	1.0850
[0.8581]	[0.5278]	[0.0027]	[0.3481]	[0.6670]	[0.5812]

Panel B. EGARCH(1,1) Estimates

a_0	a_1	a_2			
0.4442*	0.2665*	0.1605*			
(0.0872)	(0.0788)	(0.0713)			
α_0	α_1	α_2	β_1	γ	
0.0752	-0.0072	0.0848	-0.5500	-2.5057*	
(0.2217)	(0.1420)	(0.0886)	(0.3714)	(0.5365)	
LB Q **(6)**	***LB*** Q^2 **(6)**	**LR**	**Skewness**	**Kurtosis**	**Normality**
2.4032	7.9690	17.4114*	-0.2645	0.1013	2.4419
[0.8791]	[0.2403]	[0.0000]	[0.1276]	[0.7725]	[0.2949]

Note: See Table 1.

* denotes 5-percent significance level.