Modeling U.S. Historical Time-Series Prices and Inflation Using Various Linear and Nonlinear Long-Memory Approaches

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ABSTRACT

This paper estimates the complete historical US price data by employing a relatively new statistical methodology based on long memory. We consider, in addition to the standard case, the possibility of nonlinearities in the form of nonlinear deterministic trends as well as the possibility that persistence exists at both the zero frequency and a frequencies away from zero. We model the fractional nonlinear case using Chebyshev polynomials and model the fractional cyclical structures as a Gegenbauer process. We find in the latter case that that secular (i.e., long-run) persistence and cyclical persistence matter in the behavior of prices, producing long-memory effects that imply mean reversion at both the long-run and cyclical frequencies.

Keywords: Persistence, Cyclicality, Chebyshev polynomials, Gegenbauer processes

JEL Classification: C22, E3

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1. Introduction

Most of the empirical literature on long-memory models of prices and inflation has focused on the case where the singularity or pole in the spectrum occurs at the zero frequency. Different degrees of persistence, stationarity, and mean-reversion occur depending on the value of the fractional integration parameter (see, e.g., Kumar and Okimoto, 2007; Boubaker et al., 2016; Canarella and Miller, 2015, 2016a, b). In policy terms, the importance of persistence in prices and inflation stems from the economy's susceptibility to crisis and contagions as well as the possibility that exogenous shocks can produce permanent effects. Persistence of prices and inflation at frequency zero, although a dominant characteristics of these time series, however, is not the only feature of these time series.

Many macroeconomic time series, such as prices, exhibit nonstationary movement. Cases may exist, however, where persistence at frequency zero is accompanied by persistence at cyclical frequencies. One stylized fact that characterizes the economy over the business cycle is the co-movement of prices and output. It is well-known that if output movements result from demand shocks, prices are pro-cyclical; by contrast, if shocks originate from the supply side, prices are counter-cyclical. The new classical macroeconomics (Lucas, 1972, 1976) as well as Keynesian economics (Mankiw, 1989) provide evidence in support of a positive correlation between U.S. prices and output. The real business cycle theory, on the other hand, (Kydland and Prescott, 1982; Long and Plosser, 1983) support the presence of an inverse relationship between prices and output. Whether prices exhibit pro-cyclical or countercyclical movement, the need to model adequately the cyclical component of prices is well documented in the literature.

This paper focuses on persistence and cyclicality in the U.S. price level, using historical annual data that spans 1774 to 2015. The data cover the various components of the modern history of the international monetary systems, including the bimetallic standard era (1787-1873), the classical gold standard era (1873-1914), the interwar period (1915-1944), the Bretton Woods system (1945-1971), and the post-Bretton Woods system (1971-present) and, thus, provide a unique opportunity to consider how the time-series properties of U.S. prices vary across different monetary regimes and institutions. Clearly, over such a long time period, structural breaks probably have occurred between different regimes in price determination, and the empirical analysis should reflect that. Consequently, in addition to persistence and cyclicality, this paper considers the possibility that nonlinearities may characterize the behavior of US prices.

We estimate the U.S. data using a fractional integration approach, but employ a generalized definition of long-memory, which allows the inclusion of one or more singularities or poles in the spectrum at various frequencies. Specifically, we estimate U.S. prices with three classes of fractional integration I(*d*) models using the Whittle parametric function in the frequency domain (Dalhaus, 1989) along with a Lagrange Multiplier (LM) testing procedure developed by Robinson (1994), which remains valid even in nonstationary contexts.-

The first class of models considers the standard case of fractional integration at the long run or zero frequency, and captures the persistence of U.S. prices and inflation (i.e., the long-run movement at zero frequency). Recent contributions on inflation persistence in the United States that use alternative long-memory methodologies include Caporale and Gil-Alaña (2002, 2010, 2013), Gil-Alaña (2000), Kumar and Okimoto

(2007), Gadea and Mayoral (2006), Canarella and Miller (2015, 2016a, 2016b), Boubaker et al. (2016) among many others.[1]

The second class adopts a fractional integration model that incorporates nonlinear deterministic terms in the form of Chebyshev polynomials, as nonlinearities may exist in the historical data series as a result of different monetary regimes (Caporale and Gil-Alaña, 2007). Finally, the third class of long-memory models considers the possibility that the data may display two poles or singularities in the spectrum, one at the zero frequency, corresponding to the long-run behavior of prices, and another at a frequency away from zero, affecting the cyclical structure of prices (Caporale and Gil-Alaña, 2005; Gil-Alaña, 2005; Caporale and Gil-Alaña, 2014; Gil-Alaña and Gupta, 2014). In this latter case, the data may still display the property of long-memory, but the autocorrelations exhibit a cyclical structure that decays slowly. The cyclical structure is modeled as a Gegenbauer process, which produces persistent stochastic cycles.

We find that both the secular (long-run) and the cyclical components matter, and the two orders of integration differ statistically from zero and one, the long-run being more important (in terms of persistence). Shocks affecting the two components persist and revert to their means (i.e., they disappear in the long run). Nevertheless, unlike the first two classes of long-term models, the analysis in the third class of models refers

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[1] As the existing literature frequently notes, inflation persistence plays an important role in the conduct of monetary policy as well as the development of the underlying macroeconomic theories. Inflation persistence measures the speed with which the inflation rate returns to its equilibrium level after an inflationary shock. If the inflation rate returns to its equilibrium level quickly (i.e., the inflation rate exhibits less persistence) after a shock, then the monetary authorities can more effectively reduce inflation fluctuations, all else equal (Fuhrer, 1995). High inflation persistence, on the other hand, causes shocks to exert long-lasting effects and may require a strong policy response to affect the dynamics of inflation and bring it under control. In the worst case, inflation may follow a random-walk (i.e., an *I(1)*) process, making it impossible for central banks to control inflation. In the best case, inflation may follow a stationary (i.e., *I(0)*) process, implying that it reverts to its equilibrium level rapidly after a random shock. In this latter case, the response to the inflationary shock may not require an active monetary policy. Thus, the optimal timing and size of monetary policy crucially depend on not only knowledge of how shocks affect the dynamics of inflation but also on the degree of persistence that identifies the inflation process. In this regard, we note that inflation persistence plays an important role in the current debate on inflation targeting (IT). When a central bank successfully anchors inflationary expectations by its inflation targeting policy, it reduces or eliminates inflation persistence, since well-anchored inflationary expectations depend less on past inflation.

only to prices, and not inflation, since in first differences, the interaction with the cyclical component is not meaningful.

The paper's outline includes the following sections. Section 2 briefly describes the various econometric methods. Section 3 reports the results of our econometric analysis. Section 4 briefly concludes.

2. Methods

All models examined rely on the concept of long-memory or long-range dependence as opposed to the concept of short memory (i.e., *I(0)*) behavior. We can define both concepts in the time and frequency domains. For short-memory processes, the infinite sum of its autocovariances is finite in the time domain. That is,

$$\lim_{T \to \infty} \sum_{j=-T}^{j=T} \left|\gamma_j\right| < \infty.$$

In the frequency domain, short memory implies that the spectral density function, defined as follows:

$$f(\lambda) = \frac{1}{2\pi} \sum_{j'-\infty}^{j=\infty} \gamma_j \cos \lambda j, \qquad -\pi < \lambda \leq \pi,$$

is positive and finite at all frequencies on the spectrum. That is,

$$0 < f(\lambda) < \infty \quad \text{at all} \quad \lambda \in [0, \pi).$$

Short memory processes include the most common stationary process such as those based on (stationary) ARMA structures. In economics, however, it is common to find series that display a high degree of persistence which we cannot capture using ARMA models. Thus, many economic series display long-memory behavior.

Hipel and McLeod (1978) define a long-memory process, x_t, when

$$lim_{T \to \infty} \sum_{j=-T}^{j=T} \left|\gamma_j\right| = \infty .$$

In the frequency domain, long memory implies that the spectral density function includes at least one pole or singularity at some frequency λ in the interval *[0, π)*. That is,

$$f(\lambda) \rightarrow \infty, \quad as \;\; \lambda \rightarrow \lambda^{*}, \quad \lambda^{*} \in [0, \pi).$$

The empirical time-series literature usually focuses on the case where the singularity or spike in the spectrum takes place at the *0* frequency (i.e., $\lambda^* = 0$), which leads to the standard *I(d)* models of the form:

$$(1 - L)^{d} x_t = u_t, \quad t = 0, \pm 1, ..., \qquad (1)$$

where d can equal any real value, L is the lag-operator (e.g., $Lx_t = x_{t-1}$), and u_t is *I(0)*, as previously defined.

The most notorious case corresponds to $d = 1$, implying the existence of unit roots and nonstationarity. In this case, we need to transform by first differences to render the series *I(0)*. This standard practice emerged after Nelson and Plosser (1982), who found evidence of unit roots in fourteen U.S. macro series.

In general, however, the differencing of a series to achieve stationarity may, in fact, only require a fractional difference (Granger, 1980). As such, we identify the process as fractionally integrated. Then, we can expand the polynomial in the left-hand side of equation (1) in terms of its binomial expansion, such that, for all real d,

$$(1-L)^{d} = \sum_{j=0}^{\infty} \psi_j L^j = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = 1 - dL + \frac{d(d-1)}{2} L^2 - ...,$$

or

$$(1-L)^{d} x_t = x_t - d x_{t-1} + \frac{d(d-1)}{2} x_{t-2} - ...,$$

implying that we can express equation (1) as follows:

$$x_t = d x_{t-1} - \frac{d(d-1)}{2} x_{t-2} + ... + u_t. .$$

In this context, d plays an essential role, since it defines the degree of dependence of the time series. The higher the value of d is, the higher is the level of association between the observations. Granger and Joyeaux (1980), Granger (1980, 1981), and Hosking (1981) introduced these models that Baillie (1996), Gil-Alaña and Robinson (1997), and others later generalized.

In section 3, we estimate the differencing parameter d by different methods, including parametric and semiparametric ones. Moreover, we will employ a Lagrange Multiplier (LM) tests proposed by Robinson (1994) that allows x_t in equation (1) to equal the errors in a regression model of the form:

$$y_t = \beta^T z_t + x_t, \quad t = 1, 2, ..., \tag{2}$$

where y_t is the observed time series (e.g., log of US CPI), β is a *(kx1)* vector of unknown coefficients, and z_t is a set of weakly exogenous variables or deterministic terms that can include an intercept (i.e., $z_t = 1$), an intercept with a linear time trend ($z_t = (1, t)^T$), or any other type of deterministic processes.

In addition, we employ an extension of this method to the nonlinear case, replacing the linear regression in equation (2) by a nonlinear model based on Chebyshev polynomials in time. Cuestas and Gil-Alaña (2016) suggested this approach, which basically consists in replacing equation (2) by

$$y_t = \sum_{i=0}^{m} \theta_i P_{i,N}(t) + x_t; \quad t = \pm 1, \pm 2, ..., \tag{3}$$

where m gives the order of the Chebyshev polynomial $P_{i,N}(t)$, defined as,

$$P_{i,N}(t) = \sqrt{2}\cos\left[i\pi(t-0.5)/N\right]; \quad t = 1, 2, ..., N; \quad i = 1, 2, ...,$$

with $P_{0,N}(t) = 1$. Bierens (1997) uses Chebyshev polynomials in the context of unit-root testing.

Chebyshev polynomials can approximate highly nonlinear trends with rather low degree polynomials (Bierens, 1997; Tomasevic et al., 2009). From equation (3), if m = 0, the model contains only an intercept; if m = 1, it contains an intercept and a linear trend; and if $m > 1$, it becomes nonlinear, where the higher the value of m is, the higher is the nonlinear structure. The parameters $\theta_i\left(i=1,\ldots,m\right)$ are the nonlinear parameters where the significance of $m > 1$ parameters implies nonlinearity of the time series. An issue that immediately arises is the optimal value of m. Cuestas and Gil-Alaña (2016) argue that if one combines equations (1) and (3) in a single equation, standard t-tests will remain valid with an *I(0)* error term by definition. Then, the choice of m will depend on the significance of the Chebyshev coefficients.[2] Note that the model obtained by combining equations (1) and (3) is linear, and we can estimate d parametrically and test as in Robinson (1994) and Demetrescu, Kuzin, and Hassler (2008), among others (see Cuestas and Gil-Alaña, 2016).

Many macroeconomic time series display cyclical patterns. The existence of cycles in macroeconomic time series is a well-documented stylized fact since Burns and Mitchell (1946) first examined the U.S. economy. The appropriate way to model their cyclical behavior, however, remains controversial. Deterministic structures based on sine and cosine functions do not perform well in the majority of the cases. We can capture cyclical patterns through a simple AR(2) process with complex roots. In the case of high levels of persistence or even nonstationarity, however, a cyclical long-memory model can prove more appropriate. In such cases, we extend the model in equation (1) by incorporating another pole or singularity in the spectrum at a non-zero frequency.

Thus, the third model represents x_t as follows:

[2]See Hamming (1973) and Smyth (1998) for a detailed description of these polynomials.

$$(1-L)^{d_1}(1-2\cos w_r L + L^2)^{d_2} x_t = u_t, \quad t = 1, 2, \dots, \qquad (4)$$

where d_1 is the order of integration corresponding to the long-run or zero frequency, and d_2 is the order of integration with respect to the non-zero (cyclical) frequency, and u_t is an I(0) process. The second polynomial in the left hand side in equation (4) uses Gegenbauer processes, where $w_r = 2\pi r/T$ and $r = T/s$. Thus, s indicates the number of time periods per cycle, while r refers to the frequency that has a pole or singularity in the spectrum of x_t. Note that if $r = 0$ (or $s = 1$), the fractional cyclical polynomial in equation (4) becomes $(1 - L)^{2d}$, which is the polynomial associated with the long-run or zero frequency. Andel (1986) introduced this process, which Gray, Zhang and Woodward (1989, 1994), Giraitis and Leipus (1995), Chung (1996a, 1996b), Gil-Alaña (2001) and Dalla and Hidalgo (2005) among others subsequently analyzed.

We can show that by denoting $\mu = cosw_r$, for all $d_2 \neq 0$,

$$(1-2\mu L + L^2)^{-d_2} = \sum_{j=0}^{\infty} C_{j,d_2}(\mu) L^j,$$

where $C_{j,d_2}(\mu)$ are orthogonal Gegenbauer polynomial coefficients recursively defined as follows:

$$C_{0,d_2}(\mu) = 1, \qquad C_{1,d_2}(\mu) = 2\mu d_2,$$

$$C_{j,d_2}(\mu) = 2\mu\left(\frac{d_2-1}{j}+1\right)C_{j-1,d_2}(\mu) - \left(2\frac{d_2-1}{j}+1\right)C_{j-2,d_2}(\mu), \; j = 2, 3, \dots.$$

Using, once again, Robinson’s (1994) LM tests, we can test the null hypothesis:

$$H_0: \; d \equiv (d_1, d_2)^{Tr} = (d_{10}, d_{20})^{Tr} \equiv d_0, \qquad (5)$$

in equation (4) for real values $d_o = (d_{10}, d_{20})^T$, where T means transposition, and x_t are the regression errors in equation (2). The specific form of the test statistic, denoted by $\hat{R}$, is found in Gil-Alaña (2005). Under very general regularity conditions, Robinson (1994) and Gil-Alaña (2005) shows that for this particular version of his tests,

$$\hat{R} \rightarrow_d \chi_2^2, \qquad as \quad T \rightarrow \infty, \tag{6}$$

where T indicates now the sample size and "$\rightarrow_d$" stands for convergence in distribution. Thus, unlike other procedures, we now face a classical large-sample testing situation.

We reject H_0 against the alternative H_A: $d \neq d_0$, if $\hat{R} > \chi^2_{2,\alpha}$, where $Prob\ (\chi_2^2 > \chi^2_{2,\alpha}) = \alpha$. Several reasons exist for using this approach. First, this test is the most efficient in the Pitman sense against local departures from the null. That is, if we implement it against local departures of the form: H_A: $d = d_0 + \delta T^{-1/2}$, for $\delta \neq 0$, then the limit distribution is a $\chi_2^2(v)$ with a non-centrality parameter v that is optimal under Gaussianity of u_t. Moreover, we do not require Gaussianity for the implementation of this procedure, but only a moment condition of order 2.

3. Empirical results

We gather the U.S. consumer price index (CPI) data, covering the period 1774-2015, from the website of Professor Robert Sahr of Oregon State University,[3]and compute the inflation series as the first difference of the natural logarithm of the CPI, which implies that our effective sample starts from 1775.

Figure 1 shows the time-series plots of the log of CPI and the rate of inflation, along with their corresponding correlograms and periodograms. We observe first that the prices were relatively stable with some cyclical pattern until the Great Depression. After that, prices rose continuously until the present. We clearly see the nonstationary nature of the log CPI data through the correlogram, whose values decay slowly, and through the periodogram, whose highest value occurs at the smallest frequency. On the

[3] The data can be downloaded from: http://liberalarts.oregonstate.edu/spp/polisci/research/inflation-conversion-factors.

other hand, the correlogram of the inflation displays many significant values, while the periodogram also displays the highest frequency at the zero frequency. Nevertheless, this peak may hide others at a frequency away from zero.

[Insert Figure 1 about here]

The first model we examine is the standard *I(d)*. We estimate the parameters in equations (1) and (2) with $z_t = (1, t)^T$, and test the null H_0: $d = d_0$, for any real value d_o such that the model tested becomes:

$$y_t = \beta_0 + \beta_1 t + x_t; \quad (1-L)^{d_0} x_t = u_t \quad t = 1, 2, \ldots, \; . \tag{7}$$

Given the parametric nature of the test, we need to specify the functional form of the disturbance term u_t. In particular, we consider four different specifications: white noise, AR(1), AR(2), and the exponential spectral model of Bloomfield (1973). The latter is a nonparametric method to approximate ARMA structures with a few number of parameters and accommodates extremely well in fractional integration contexts (see, e.g., Gil-Alaña, 2004).

[Insert Table 1 about here]

Table 1 displays the estimates of d along with the 95% confidence intervals of the non-rejection values of d_0 in equation (7) for both the log CPI and the inflation rate, and for the three standard cases examined in the literature of no regressors (i.e., $\beta_0 = \beta_1 = 0$*a priori* in equation (7)): an intercept (β_0 unknown and $\beta_1 = 0$*a priori*); and an intercept with a linear time trend (β_0 and β_1 unknown). The bolded entries in the table correspond to the most adequate specification for the deterministic terms, which according to the t-values of these coefficients (unreported), is the intercept-only case. If u_t is white noise or follows the model of Bloomfield, then the estimated d exceeds 1 and the unit-root null hypothesis ($d = 1$) is, in fact, rejected in favor of the alternative of $d > 1$. Using AR components, however, we cannot reject the unit-root hypothesis, even though the estimated d still exceeds 1. Due to the disparity in these results, we also conducted a semi-parametric approach (Robinson, 1995), though we do not impose a functional form on the *I(0)* disturbances term.

[Insert Figure 2 and Table 2 about here]

Figure 2 displays the estimates of d taking into account all the bandwidth values from $m = 2, ..., T/2$. We observe that for small bandwidth values, the estimated values of d lie within the *I(1)* interval, however, for large bandwidths, the values of d are significantly above 1. Table 2 displays the specific values from $m = 10$ *to* 20 ($m^{0.5} = 15.55$). We cannot reject the unit-root null hypothesis of $d = 1$in any single case.

The second model considers the possibility of nonlinear deterministic terms. For this purpose, we use the Chebyshev polynomials in time as presented in the previous section. Thus, the estimated model is now:

$$y_t \;=\; \sum_{i=0}^{3} \theta_i P_{iN}(t) \;+\; x_t; \quad (1-L)^{d_o} x_t \;=\; u_t \qquad t \;=\; 1,\, 2,\, \ldots, \qquad (8)$$

[Insert Table 3 about here]

We examine the cases of uncorrelated (white noise) and autocorrelated (Bloomfield-type) errors. The results prove consistent in terms of the degree of integration. The estimated value of d equals 1.27 in case of the log CPI data, and 0.27 for the inflation rate with white noise errors. These values are slightly smaller (1.12 and 0.11) for the Bloomfield-type disturbances and we cannot reject the unit-root null in these two cases. More importantly, we find evidence of nonlinearity in only a single case, corresponding to the inflation rate with white-noise errors.[4]

Finally, in the third model, we incorporate the possibility of cyclicality. Here, we consider a model of the following form:

[4] Using other types of nonlinear deterministic terms such as Hermite polynomials, we do not observe any evidence of nonlinearities in the data.

$$y_t = \beta_0 + \beta_1 t + x_t; \quad (1-L)^{d_{1o}}(1 - 2\cos w_r L + L^2)^{d_{2o}} x_t = u_t \qquad (9)$$

and examine once more the three cases of no regressors, an intercept, and an intercept with a linear time trend, for the four cases of white noise, AR(1), AR(2) and Bloomfield-type disturbances. Table 4 displays the results.

[Insert Table 4 about here]

We first observe that all the values of *j* (corresponding to the number of periods per cycle) fall between 5 and 13, which corresponds with the literature on business cycles. Moreover, except in the case of the AR(2) model, for the remaining models, the values of d_1 significantly exceed 1 with d_2 close to 0. Performing several tests based on the t-values of the deterministic terms and diagnostic tests carried out on the residuals, the most appropriate model uses AR(2) disturbances with a linear time trend.

Thus, the estimated model is as follows:

$$\underset{(14.294)}{y_t = 1.95497} + \underset{(12.629)}{0.01182t} + x_t; \quad (1-L)^{0.54}(1 - 2\cos w_{T/6} L + L^2)^{0.21} x_t = u_t$$

$$u_t = 0.542 u_{t-1} + 0.375 u_{t-2} + \varepsilon_t ,$$

with the *t*-values in parenthesis.

These findings clearly indicate that both the secular (i.e., the long-run) and the cyclical components matter. The two orders of integration differ statistically from zero and one, and the long-run order of integration appears more important (in terms of persistence). Shocks affecting the two components persist and revert to their means (i.e., they disappear in the long run).

We observe that in this case, the analysis can only refer to the log prices and not to inflation. That is, no direct way exists to derive the secular and cyclical persistence of inflation from the corresponding values of the persistence of prices. For inflation, we should conduct the analysis based on (1-*L*)logprices. But if we take the first differences, the interaction with the cyclical component possesses no meaning, as the cyclical component disappears. Thus, the results imply that the two components matter only in the behavior of the (log) prices, and produce long-memory mean-reverting effects.

4. Concluding remarks

This paper analyzes the complete historical US price data (1774-2015) using a variety of model specifications that incorporate the concept of long memory, persistence, nonlinearity, and cyclicality. We estimate U.S. prices with three classes of fractional integration I(*d*) models using the Whittle parametric function in the frequency domain (Dahlhaus, 1989) along with the testing procedure developed by Robinson (1994). We consider, in addition to the well-known linear specifications at zero frequency, the possibility of nonlinearities in the form of nonlinear deterministic trends as well as the possibility that persistence exists at both the zero frequency and a frequencies away from zero. We model the fractional nonlinear case using Chebyshev polynomials and model the fractional cyclical structures as a Gegenbauer process. We find evidence of nonlinearity in only a single case, corresponding to the inflation rate with white-noise errors.

The most important contribution of the paper, however, consists in the determination of persistence at frequencies away from zero. We find in this case that that the secular (i.e., long-run) persistence coexists with the cyclical persistence, and shocks have the long-memory effects that are mean-reverting at both the long-run and cyclical frequencies**.** We find the two orders of fractional integration differ statistically from zero and one, with the secular order of fractional integration being higher and, consequently more important in terms of persistence, than the cyclical order.

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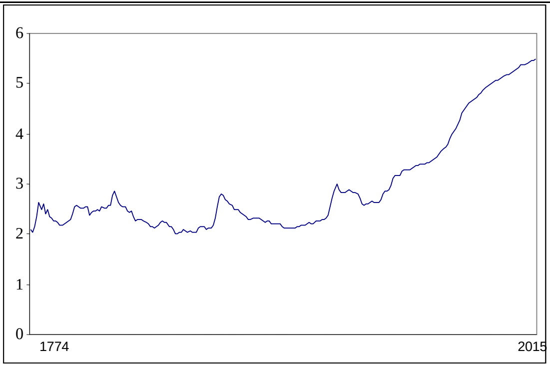
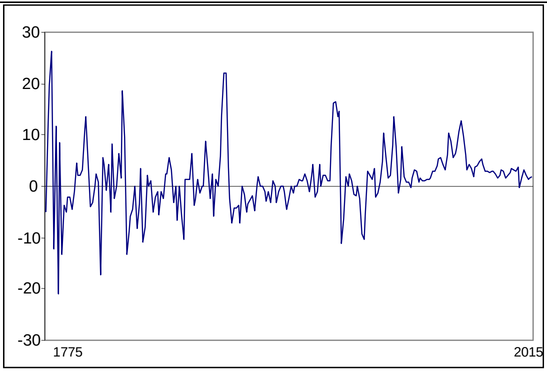
Figure 1: Time series plots

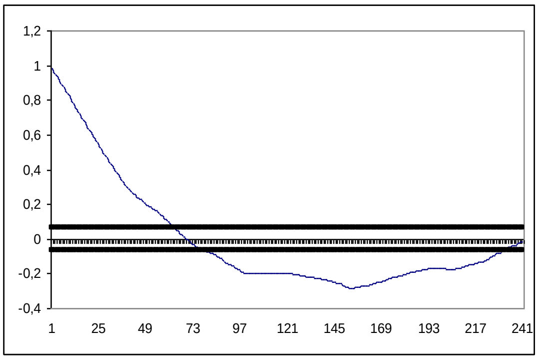
Log of CPI



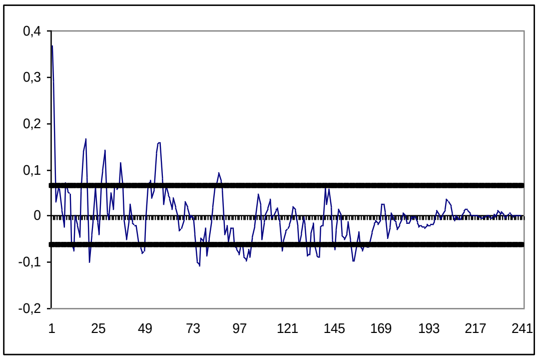
Inflation rate



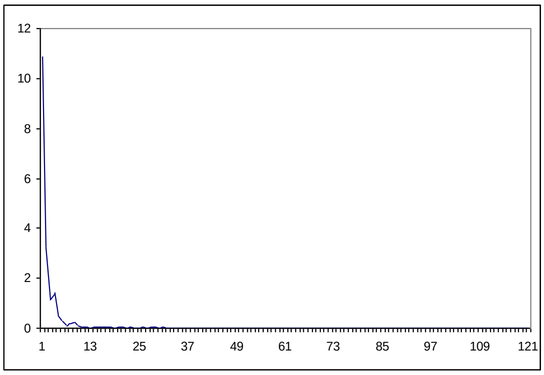
Correlogram of Log of CPI



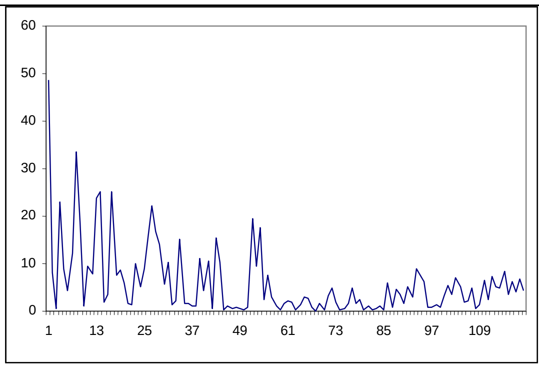
Correlogram of Inflation rate



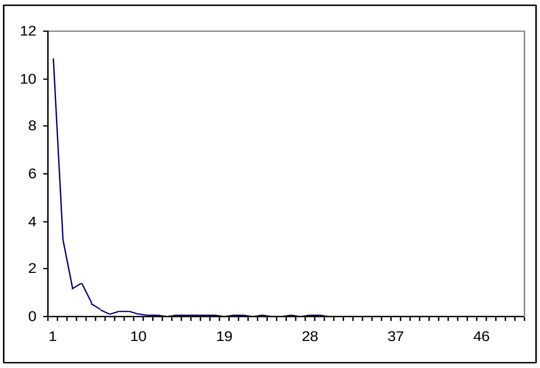
Periodogram of Log of CPI



Periodogram of Inflation rate



First 50 values Periodogram of Log of CPI



First 50 values Periodogram of Inflation

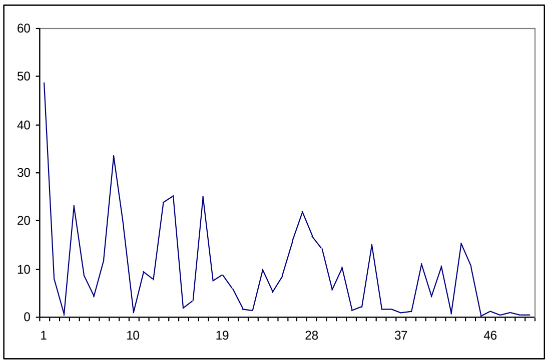
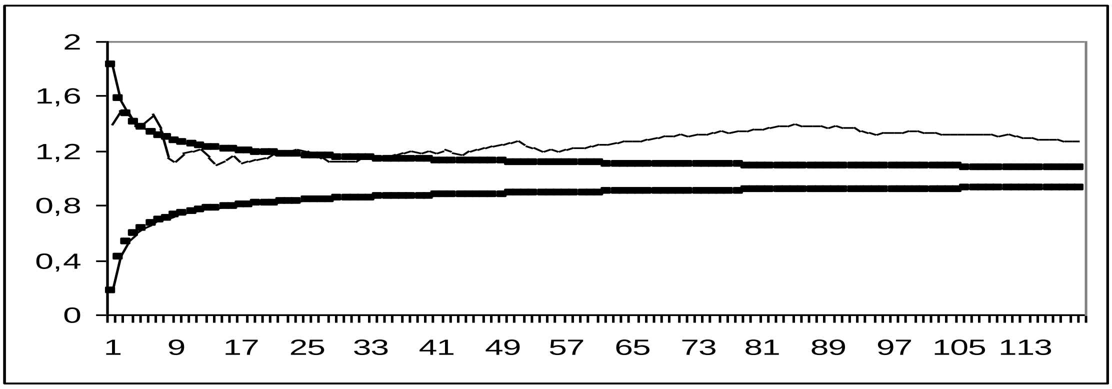


Table 1: Whittle estimates of *d* and Robinson's (1994) test results

i) Log of CPI			
	No regressors	An intercept	A linear time trend
White noise	1.06 (0.99, 1.15)	**1.29 (1.20, 1.41)**	1.29 (1.20, 1.41)
AR (1)	1.41 (1.26, 1.59)	**1.13 (0.92, 1.47)**	1.15 (0.91, 1.48)
AR (2)	1.92 (1.71, 2.14)	**1.02 (0.85, 1.31)**	1.02 (0.82, 1.32)
Bloomfield type	1.13 (1.00, 1.33)	**1.21 (1.08, 1.41)**	1.22 (1.09, 1.42)
ii) Inflation			
	No regressors	An intercept	A linear time trend
White noise	0.29 (0.20, 0.41)	**0.29 (0.20, 0.41)**	0.28 (0.18, 0.41)
AR (1)	0.13 (-0.08, 0.49)	**0.15 (-0.01, 0.48)**	0.16 (-0.02, 0.48)
AR (2)	0.01 (-0.14, 0.31)	**0.01 (-0.15, 0.32)**	0.01 (-0.14, 0.33)
Bloomfield type	0.21 (0.08, 0.42)	**0.21 (0.09, 0.41)**	0.14 (-0.03, 0.40)

Notes: In bold, the significant models according to the deterministic terms. In parenthesis the 95% confidence band of non-rejection values of d using Robinson's (1994) parametric approach.

Figure 2: Estimates of *d* based on a semiparametric method (Robinson, 1995)



Notes: In bold lines, the 95% confidence of the I(1) hypothesis (i.e., d = 1).

Table 2: Robinson's (1995) estimates of *d*

m	d	Lower 5%	Upper 5%
10	1.186*	0.739	1.260
11	1.194*	0.752	1.247
12	1.206*	0.762	1.237
13	1.143*	0.771	1.228
14	1.099*	0.780	1.219
15	1.133*	0.787	1.212
16	1.160*	0.794	1.205
17	1.115*	0.800	1.199
18	1.126*	0.806	1.193
19	1.133*	0.813	1.188
20	1.147*	0.816	1.184

m indicates the bandwidth number.

Table 3: Estimates of the nonlinear coefficients and *d* using Cuestas and Gil-Alaña (2016)

i) Log of CPI					
	d (95% interval)	θ_1	θ_2	θ_3	θ_4
Wh. Noise	1.27 (1.17, 1.40)	2.3655 (1.42)	-0.5335 (-0.50)	0.5413 (1.37)	-0.2069 (-0.87)
Bloomfield	**1.12 (0.95, 1.28)**	**2.6075 (3.11)**	**-0.6920 (-1.33)**	**0.5557 (2.46)**	**-0.2423 (-1.69)**
ii) Inflation					
	d (95% interval)	θ_1	θ_2	θ_3	θ_4
Wh. Noise	0.27 (0.15, 0.49)	1.5102 (1.01)	-1.1386 (-1.06)	0.7216 (0.75)	0.4944 (0.75)
Bloomfield	0.11 (-0.13, 0.37)	1.4252 (2.80)	-1.2558 (-2.22)	0.6940 (1.28)	0.4345 (0.83)

Table 4: Estimates of the long-run and cyclical persistence parameters in the model given by equation (4)

	Det. terms	j	d_1	d_2
White noise	No terms	5	1.34 (1.23, 1.46)	-0.05 (-0.10, 0.09)
	An intercept	7	1.29 (1.21, 1.39)	0.00 (-0.07, 0.08)
	A linear trend	7	1.29 (1.21, 1.47)	0.01 (-0.07, 0.08)
AR(1)	No terms	8	1.33 (1.02, 1.60)	0.02 (-0.29, 0.26)
	An intercept	9	1.19 (1.12, 1.37)	0.07 (-0.04, 0.34)
	A linear trend	9	1.26 (1.14, 1.40)	0.21 (-0.01, 0.37)
AR(2)	No terms	6	0.78 (0.69, 0.93)	-0.35 (-0.41, -0.29)
	An intercept	6	0.92 (0.83, 1,05)	-0.44 (-0.58, -0.32)
	A linear trend	**6**	**0.54 (0.27, 0.83)**	**0.21 (0.04, 0.41)**
Bloomfield	No terms	13	1.48 (1.11, 1.54)	-0.06 (-0.38, 0.14)
	An intercept	9	1.19 (1.03, 1.40)	0.07 (-0.32, 0.16)
	A linear trend	8	1.24 (1.10, 1.43)	0.09 (-0.08, 0.25)

Notes: In bold, the selected model across the different specifications.